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ORIGIN OF THE INFLATIONARY UNIVERSE[1]

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Abstract

We give a consistent description of how the inflationary Universe emerges in quantum cosmology. This involves two steps: Firstly, it is shown that a sensible probability peak can be obtained from the cosmological wave function. This is achieved by going beyond the tree level of the semiclassical expansion. Secondly, due to decoherence interference terms between different semiclassical branches are negligibly small. The results give constraints on the particle content of a unified theory.

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The standard big-bang model provides a successful scenario for the evolution of our Universe. Although there is still some uncertainty about the very early phase, the idea that the Universe underwent a period of exponential ("inflationary") expansion at an early time (about 10^{-33} sec after the big bang) is very promising. Not only does such a scenario avoid some of the shortcomings of the standard model, it can also give quantitative predictions for structure formation. In fact, all observed structure in the Universe can be traced back to quantum fluctuations during the inflationary era. The predictions are consistent with the anisotropy spectrum of the cosmic microwave background radiation observed by the COBE-satellite and earth-based telescopes.

What is the origin of inflation? Usually, the *no-hair conjecture* is invoked [1, 2]. This conjecture states that for a positive (effective) cosmological constant, a general spacetime approaches locally a De Sitter metric for asymptotically late times. The positive cosmological constant is assumed to arise from particle physics and is here taken for granted. The implicit assumption in this conjecture is that scales originally smaller than the Planck length do not affect the expansion. This is, however, an assumption about the unknown quantum theory of gravity.

This situation has led to the general belief that the origin of inflation can only be understood within *quantum* cosmology. In the following we shall attempt to give a precise description of this quantum origin. This might not be the final answer, but yields a consistent picture that is based on present knowledge. We shall, in particular, give answers to the questions: Can one calculate from quantum cosmology the *probability* of inflation? Moreover, can one quantitatively understand how the transition from the quantum era to the classical evolution proceeds?

One might wonder whether these questions can only be answered after a final quantum theory of gravity – perhaps superstring theory – will be available. It is, however, generally assumed that inflation occurs at a scale about five orders of magnitude below the Planck scale. This is also indicated by the size δ of the anisotropies in the spectrum of the cosmic microwave background, $\delta \approx 10^{-5}$. According to the inflationary scenario, this should reflect the fact that $m_I/m_P \approx 10^{-5}$, where m_I is the energy scale of inflation and $m_P = \sqrt{\hbar c/G}$ is the Planck mass. Since genuine quantum-gravitational effects are of the order $(m_I/m_P)^2 \approx 10^{-10}$ [3], an effective theory of quantum gravity should be sufficient to calculate effects at this scale. As the classical limit at such scales should be general relativity, the effective theory should be canonical quantum gravity to an excellent approximation.

The central quantity in canonical quantum cosmology is the wave function $\Psi(a, \varphi, f)$, where a is the scale factor, φ is the field causing inflation (the "inflaton"), and f denotes all other degrees of freedom. The wave function obeys the Wheeler-DeWitt equation and does not contain any information about a classical time parameter t. Such a time parameter can, however, be recovered, $\Psi(a, \varphi, f) \to \Psi_t(\varphi, f)$, in a semiclassical approximation with respect to m_I/m_P [3]. Since this quantity is small in our case, the validity of this approximation

should be excellent. Then a and φ are not independent, but related through the semiclassical time t; we could alternatively use a as an argument of $\Psi_t(\varphi, f)$.

The semiclassical approximation can be performed either as a WKB limit for the Wheeler-DeWitt equation or as a saddle-point limit on the path integral. In both ways one finds an approximate classical spacetime on which quantum degrees of freedom propagate. To recover an inflationary Universe in the classical limit, this spacetime is taken to be an approximate *De Sitter* space. But what boundary conditions should one use? Already Albert Einstein had to deal with the problems of specifying boundary conditions, although in ordinary space: "If it were possible to consider the world as a continuum that is closed in all spatial directions, no such boundary conditions would be necessary at all".[2] In quantum cosmology, a similar idea is contained in the so-called no-boundary proposal [5]: One should sum in the path integral only over such manifolds that have *no* initial boundary. In the semiclassical approximation, this gives rise to the *De Sitter instanton*: Classical De Sitter space is attached to half of a euclidean four-sphere with radius

$$H^{-1}(\varphi) \equiv \frac{3m_P^2}{8\pi V(\varphi)} , \tag{1}$$

where $V(\varphi)$ is the inflationary potential, and H is the Hubble parameter.

There also exist other sensible boundary conditions including the so-called tunneling one [6]. Although they do not have such a geometrical interpretation, in the semiclassical approximation they just correspond to the choice of a different WKB solution.

To discuss both the probability for inflation and the emergence of classical properties, the reduced density matrix for scale factor and inflaton should be investigated. This density matrix is calculated from the full quantum state upon integrating out the degrees of freedom f,

$$\rho_t(\varphi, \varphi') = \int \mathcal{D}f \ \Psi_t^*(\varphi', f)\Psi_t(\varphi, f) . \tag{2}$$

To calculate the probability one has to set $\varphi' = \varphi$. In earlier work, the saddle-point approximation was only performed up to the highest, tree-level, approximation [5, 6]. This yields

$$\rho(\varphi, \varphi) = \exp[\pm I(\varphi)] , \tag{3}$$

where $I(\varphi) = -3m_P^4/8V(\varphi)$. The lower sign corresponds to the no-boundary condition, while the upper sign corresponds to the tunneling condition [6]. The problem with (3) is that ρ is not normalisable: Scales $m_I \equiv H(\varphi) > m_P$ contribute significantly and thus spoil the original idea of the no-hair conjecture. Results based on tree-level approximations can thus not be trusted.

[2] "Wenn es nämlich möglich wäre, die Welt als ein nach seinen räumlichen Erstreckungen geschlossenes Kontinuum anzusehen, dann hätte man überhaupt keine derartigen Grenzbedingungen nötig." [4]

The situation is considerably improved if loop effects are taken into account [7, 8, 9]. They are incorporated by the loop effective action Γ_{loop} which is calculated on the De-Sitter instanton. In the limit of large φ (that is relevant for investigating normalisability) this yields in the one-loop approximation

$$\Gamma_{loop}(\varphi)|_{H\to\infty} \approx Z \ln \frac{H}{\mu} \ , \tag{4}$$

where μ is a renormalisation mass parameter, and Z is the anomalous scaling. Instead of (3) one has now [7]

$$\begin{aligned} \rho(\varphi,\varphi) &\approx H^{-2}(\varphi) \exp\left(\pm I(\varphi) - \Gamma_{loop}(\varphi)\right) \\ &\approx \exp\left(\pm \frac{3m_P^4}{8V(\varphi)}\right) \varphi^{-Z-2} \ . \end{aligned} \tag{5}$$

This density matrix is normalisable provided $Z > -1$. This in turn leads to reasonable constraints on the particle content of the theory [7, 8].

One can easily obtain a probability peak for the initial value of the inflaton, φ_I, at the onset of inflation in a tunneling model with nonminimal coupling [8, 9]: $\varphi_I \approx 0.03 m_P$, with a dispersion $\Delta\varphi \approx 10^{-7} m_P$, and a corresponding Hubble parameter $H(\varphi_I) \approx 10^{-5} m_P$. The relative width

$$\frac{\Delta\varphi}{\varphi_I} \approx \frac{\Delta H}{H} \approx 10^{-5} \tag{6}$$

corresponds to the observed anisotropies in the cosmic microwave background. This approach also works for open inflation with no-boundary initial condition [10]. We emphasise that no anthropic principle is needed to obtain these results, in contrast to earlier tree-level calculations.

Since quantum theory by itself does not yet yield classical ensembles, it would be premature to interpret the above results as giving by itself the probabilities for inflationary universes with different Hubble parameters. This can only be done after a quantative understanding of the quantum-to-classical transition has been gained. How can this be achieved?

It is now quite generally accepted that classical properties for a subsystem emerge from the irreversible interaction of this quantum system with its natural environment – a process called *decoherence* [11]. But what constitutes system and environment in quantum cosmology where there is no external measuring agency? It was suggested in [12, 13] to consider the global degrees of freedom a and φ as the "relevant" variables which are decohered by "irrelevant" degrees of freedom such as density fluctuations, gravitational waves, or other fields. The latter comprise the variables f that are integrated over in (2).

Since information about interference terms is contained in the non-diagonal elements of the reduced density matrix, one has to evaluate (2) for $\varphi \neq \varphi'$ (or alternatively, $a \neq a'$, which we use for illustration below). The resulting expressions are, however, ultraviolet-divergent and must be regularised. They also depend on

the parametrisation of quantum fields. This was investigated in detail for bosons in [14] and for fermions in [15]. Unfortunately, standard regularisation schemes such as dimensional regularisation do not work since they spoil a crucial property of the density matrix – its boundedness. We have thus put forward the physical principle that decoherence should be absent in the absence of particle creation since decoherence is an irreversible process. There should, in particular, be no decoherence for static spacetimes. This has guided us to use a certain conformal reparametrisation for bosonic fields and a certain Bogoliubov transformation for fermionic fields, which renders the decoherence effects finite.

We have calculated for the above semiclassical De Sitter background the decoherence factor $D_t(\varphi, \varphi')$ that multiplies the expression for the reduced density matrix in the absence of interactions. Because there is no particle creation if the "environmental" fields are massless and conformally invariant, $D_t(\varphi, \varphi') = 1$ and decoherence is absent. This is no longer true for other fields. Taking a massive scalar field with mass m, for example, one finds for the absolute value of the decoherence factor in the a-representation

$$|D_t(a, a')| \approx \exp\left(-\frac{\pi m^3 a}{128}(a - a')^2\right) \ . \tag{7}$$

Interference terms between different $a = \cosh Ht/H$ and $a' = \cosh H't/H'$, $H = H(\varphi)$, $H' = H(\varphi')$, are therefore suppressed, and the effect becomes more efficient with increasing t. The result for gravitons is similar, with the mass m being replaced by the Hubble parameter H. For massive fermions, the power of the mass is m^2 instead of m^3.

It becomes clear from these examples that the Universe acquires classical properties after the onset of the inflationary phase, and it makes sense to speak of a probability distribution for classical universes. "Before" this phase, the Universe was in a timeless quantum state which does not possess any classical properties. Viewed backwards, different semiclassical branches would meet and interfere to form this timeless quantum state. After the background has become classical, the stage is set for other degrees of freedom to assume classical properties, such as for the primordial fluctuations that arise in the inflationary Universe and lead to the observed anisotropies of the microwave background radiation [16].

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